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《論　　説》

Does the “Law of the Stronger” Hold?: A Theory of Tax Shifting with Bargaining Power*

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Abstract

Shifted tax is usually identified with a change in price caused by taxation. The price change csused by taxation, however, contains a *reasonable* change and an additional change. The latter one is the shifted tax. We introduce the terminology *net tax shifting* which captures the pure shifting effect of taxation. Tax shifting is often considered to reflect a kind of economic power. We explain tax shifting as a result of an imbalance of bargaining power. Mering (1942) noted an influential view that those who are in an economically stronger position will struggle to thrust the tax burden upon the weaker: the “law of the stronger.” It seems that the “law of the stronger” still survives, as Mering (1942) also suggested that this idea seems to correspond closely with common sense. We investigate

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whether this law holds or not and acquire several negative results. Tax shifts *from the weaker to the stronger*.

Keywords: tax shifting, bargaining power, law of the stronger.
JEL classification number: D23, H22, H26, L14.

1 Introduction

We consider a situation where two firms are related vertically and bargain over the price and quantity of goods. The downstream firm resells the goods to consumers. When a tax is levied on one of them, it shifts to others either fully, partially or not at all. We show that bargaining power determines tax shifting. There is an influential view that tax shifts from the stronger to the weaker with respect to bargaining power. The "law of the stronger" was introduced in Mering (1942, p.17):[1]

> First, there is a view which seems to correspond most closely to common sense. As such it is often encountered in popular treatments of the problem but it may also be found in scientific writings. It is without doubt suggested in the work of Adolf Wagner and implied not infrequently in the deductions of Seligman. It is the view that those who are in an economically stronger position will emerge as victors in the struggle to escape incidence of the tax.

This law seems to be plausible, however, theoretical validity has not been showed sufficiently. We examine whether this law holds in the model.

Seligman (1927), as well as Musgrave (1959), formerly stated that the shifting of a tax was the process of a transfer of tax, and avoided distinguishing the

1 Mering (1942) declared against the "law of the stronger" and stated that there is no general rule regarding the extent to which the shifting of taxation is influenced by economic bargaining power.

shifted tax itself exactly. Mering (1942) admitted that to define tax shifting precisely was too difficult. Musgrave (1959) stated that the difference between the impact incidence and the effective incidence might be referred to as the result of shifting. Musgrave (1959) did not focus on tax shifting but on *tax incidence*.[2] After Harberger (1962), many studies on tax incidence have been completed (for example, Atkinson and Stiglitz (1980); Bhatia (1986)). The price change caused by taxation is the main subject in incidence analysis. Tax incidence, however, is not equivalent to tax shifting. There have been few studies on tax shifting.

Researchers usually use the difference between the pre-tax price and the post-tax price to measure *tax shifting*. Stiglitz (2000) defines *shifted tax* as the ratio of the change in equilibrium price to the change in tax in partial equilibrium settings. Except for slight differences, the price change itself that results from taxation is usually identified with shifted tax (for example, Eagly (1983); Lockwood (1990); McCorrison, Morgan and Rayner (1990); Narayanan (1989)).

Suppose, for example, that an excise tax t is levied on a monopolist for each unit of production in an ordinary partial equilibrium setting (a down-sloping demand curve and an up-sloping marginal cost curve). The price increases and the quantity decreases. Let p and $\hat{p}$ be the pre-tax and the post-tax prices respectively. It is usually said that the tax *partially* shifts *forward*. "Partially" means "$\hat{p} - p < t$" and "forward" means "from the producer to consumers". This view identifies $\hat{p} - p$ with the shifted tax. If you ask the producer, "Do you shift tax to consumers?", the producer may reply, "No, the rise in price results from the decrease in production. This is not tax shifting. The price $\hat{p}$ is *reasonable* for the decreased production level whether the tax rate changes or not."

2 According to Musgrave (1959), the effective incidence is the actual change in distribution that results as a given tax is imposed or tax substitution is made. The impact incidence is the change that would result if the income position of a new taxpayer were reduced by the amount of tax addition, or the income position of a former taxpayer were improved by the amount of tax remission, while the positions of all others remained unchanged.

Seligman (1927, p.1) stated, "Thus the person who originally pays the tax may not be the one who bears its burden in last instance. The process of the transfer of a tax is known as the *shifting* of the tax, while the settlement of the burden on the ultimate taxpayer is called the *incidence* of the tax." Traditionally, tax shifting is referred to as the transfer of the payment of a tax. We need to distill the pure shifted tax from the change in equilibrium price to capture this concept. A price change caused by taxation can contain shifted tax and a reasonable price change which reflects the change in the producer's reasonable receipt and consumers' reasonable payment. We cannot regard the price change as shifted tax since the reasonable price can be affected by the prices and allocation of all commodities.

In view of this traditional terminology, tax shifts *onward* from a maker to a wholesaler, from a wholesaler to a retailer, and from a retailer to consumers. A sufficient explanation of the direction and degree of shifting in supply chain settings, however, has not been offered in the literature. Lockwood (1990) studies the effect of taxation on wage rates in a unionized labor market. McCorrison, Morgan and Rayner (1990) investigates the effect of a tax levied on intermediate goods (final goods, respectively) on final goods price (intermediate goods price, respectively) in a model with two vertically related industries where the upstream is competitive and the downstream is oligopolistic. We combine an *ultimatum bargaining* structure with a typical supply chain setting (a maker, a wholesaler, a retailer, and consumers)[3] to examine the relationship between bargaining power and tax shifting.

The remainder of this paper is organized as follows. We set up the model in Section 2. Section 3 provides several uniqueness properties of equilibrium. We examine tax incidence in Section 4. Section 5, the main section, suggests a new definition for tax shifting and examines tax shifting in the model. We show that

3 We use a model with two firms for simplicity.

the “law of the stronger” does *not* hold. Section 6 provides some concluding remarks. Most of the proofs are collected in the Appendix.

2 Model

Suppose that a *supply chain* consists of two vertically related firms, M and R. Let M be the upstream firm (maker) and R be the downstream firm (retailer). R uses an intermediate input provided by M to produce final goods, and behaves as a monopolist in the final goods market. Let $P = a - bX$ be the inverse final goods demand function, where $P \geq 0$ and $X \geq 0$ denote the price and quantity demanded respectively, and both a and b are positive constant parameters. M cannot produce final goods. Let $C_M(x)$ and $C_R(X)$ be the production costs for M and R respectively, where x is M’s output and X is R’s output. We do not distinguish R’s output from sales since they are equivalent in equilibrium. M’s output is also not distinguished from sales for the same reason.

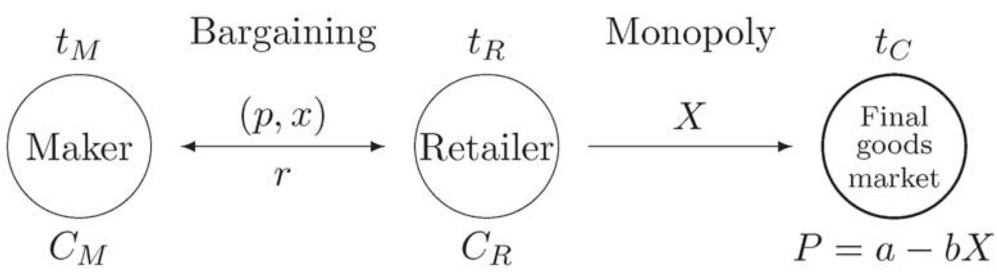

Figure 1: The supply chain

The firms bargain over the terms of intermediate trade. We introduce a simple bargaining structure, *ultimatum bargaining*, where one firm proposes a price quantity pair and the other responds yes (Y) or no (N). If the responder accepts an offer (p, x), M produces x and gets the payoff $px - C_M(x) - t_M x$, where t_M is the tax rate imposed on M for selling intermediate goods. Then R produces X to sell at the monopoly price $P = a - bX - t_C$, where t_C is the tax rate imposed on consumers (C) for buying final goods. R gets the payoff $PX - C_R(X) - px - t_R X$, where t_R is the tax rate imposed on R for selling final goods. It is natural to presume that $x \geq X$.[4] If the responder rejects an offer, no final goods are supplied and every firm gets a payoff of 0. The game, *supply chain game* (SCG), proceeds as follows (see Figure 1):

Period 1: One firm proposes (p, x) to the other.

Period 2: The firm that did not propose responds $r \in \{Y, N\}$. If $r = N$, the game is over, and each firm gets a payoff of 0. If $r = Y$, the game continues to Period 3.

Period 3: M produces x and pays the tax $t_M x$, and the transaction is made between firms. Next, R produces X, sells it in the market, and pays the tax $t_R X$. Consumers pay the tax $t_C X$.

For the remainder of the paper, we refer to the case where the direction of the offer is downstream as *forward proposing* and the case where the direction of the offer is upstream as *backward proposing*.

3 Equilibrium

We define an equilibrium strategy profile as a backward induction solution. We consider only pure strategies for simplicity, and assume that information is complete and perfect. In addition, we make the following two assumptions.

Assumption 1: For all $j \in \{M,R\}$, the production cost function $C_j(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ is continuous, twice differentiable on $\mathbb{R}_{++}$, and $C_j(0) = 0$. The marginal cost function $C'_j(\cdot)$ is non-negative and non-decreasing for all j.

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4 We can interpret R as a retailer who buys x and resells it to consumers. R has a *Leontief* production function $X = \min\{F(L, K), x\}$, where L and K are other inputs.

Assumption 2: Final goods demand is sufficiently large or marginal costs and taxes are sufficiently small, that is,

$$a - \sum_{i=\mathrm{M,R,C}} t_i > \lim_{z \to 0} \sum_{j=\mathrm{M,R}} C_j'(z).$$

Assumption 2 ensures that final goods supply is positive in equilibrium. If demandis too small or the marginal cost is too large, firms do not produce initially. If the taxes are too large, firms stop producing when the taxes are levied on them. Assumption 2 eliminates trivial equilibria with production of 0.

3.1 SCG with forward proposing

If M is the proposer in Period 1 of bargaining, M has full bargaining power and R has none at all. In Period 3, R's problem is

$$\max_{X \in [0,x]} \left(a - bX - \sum_{i=\mathrm{R,C}} t_i \right) X - C_R(X) - px$$

since R is the monopolistic final goods supplier. Therefore, R's optimal final goods supply strategy is

$$X^* = \begin{cases} \overline{X} & \text{if } x \geq \overline{X} \\ x & \text{if } x < \overline{X} \end{cases},$$

where $\overline{X}$ satisfies $a - 2b\overline{X} - \sum_{i=\mathrm{R,C}} t_i - C_R'(\overline{X}) = 0$. Assumptions 1 and 2 ensure that $\overline{X}$ is unique and positive.

R's payoff becomes $(a - bX^* - \sum_{i=\mathrm{R,C}} t_i)X^* - C_R(X^*) - px$ if R accepts M's offer (p, x) in Period 2. The set of R's optimal response strategies is

$$R_f^* = \{r : r \text{ satisfies } (1)\},$$

where

$$r = \begin{cases} \text{Y} & \text{if } (a - bX^* - \sum_{i=\text{R,C}} t_i)X^* - C_R(X^*) - px \geq 0 \\ \text{N} & \text{if } (a - bX^* - \sum_{i=\text{R,C}} t_i)X^* - C_R(X^*) - px \leq 0 \end{cases}. \quad (1)$$

Therefore, M's payoff is $(p - t_M)x - C_M(x) \leq (a - bX^* - \sum_{i=\text{R,C}} t_i)X^* - C_R(X^*) - C_M(x) - t_M x$ if R accepts M's offer.

Equilibrium strategy profiles are plural since R_f^* is not a singleton. However, the path is unique.[5]

Theorem 1: *In a SCG with forward proposing, the path* $((\mathbf{p}^*, \mathbf{x}^*), \mathbf{r}^*, (\mathbf{P}^*, \mathbf{X}^*))$ *such that:*

$$\begin{aligned} &\textit{Proposal:} && (\mathbf{p}^*, \mathbf{x}^*) = \left(a - b\overline{x} - \textstyle\sum_{i=\text{R,C}} t_i - \frac{C_R(\overline{x})}{\overline{x}}, \overline{x}\right), \\ &\textit{Response:} && \mathbf{r}^* = \text{Y}, \\ &\textit{Supply:} && (\mathbf{P}^*, \mathbf{X}^*) = (a - b\overline{x} - t_C, \overline{x}), \end{aligned}$$

is the unique equilibrium path where $\overline{x}$ *satisfies* $a - 2b\overline{x} - \sum_{i=\text{M,R,C}} t_i - \sum_{j=\text{M,R}} C_j'(\overline{x}) = 0.$

Proof: See Appendix. □

3.2 SCG with backward proposing

If R is the proposer, R has full bargaining power and M has none at all. In Period 3, R's optimal final goods supply strategy is X^*. M gets the payoff $px - C_M(x) - t_M x$ if M accepts an offer (p, x) in Period 2. The set of M's optimal response strategies is

$$R_b^* = \{r : r \text{ satisfies } (2)\},$$

5 Note that the uniqueness of the equilibrium path is not obvious. Indeed, in the case of more than two firms, the path is not necessarily unique.

where

$$r = \begin{cases} \text{Y} & \text{if } px - C_M(x) - t_M x \geq 0 \\ \text{N} & \text{if } px - C_M(x) - t_M x \leq 0 \end{cases}. \tag{2}$$

Therefore, R's payoff is $PX - C_R(X) - t_R X - px \leq (a - bX^* - \sum_{i=\text{R,C}} t_i)X^* - C_R(X^*) - C_M(x) - t_M x$ if M accepts R's offer.

Theorem 2: *In a SCG with backward proposing, the path* $((\mathbf{p}^{**}, \mathbf{x}^{**}), \mathbf{r}^{**}, (\mathbf{P}^{**}, \mathbf{X}^{**}))$ *such that:*

$$\begin{aligned} &\textit{Proposal:} \quad (\mathbf{p}^{**}, \mathbf{x}^{**}) = \left(\frac{C_M(\overline{x})}{\overline{x}} + t_M, \overline{x}\right), \\ &\textit{Response:} \quad \mathbf{r}^{**} = \text{Y}, \\ &\textit{Supply:} \quad (\mathbf{P}^{**}, \mathbf{X}^{**}) = (a - b\overline{x} - t_C, \overline{x}), \end{aligned}$$

is the unique equilibrium path where $\overline{x}$ satisfies $a - 2b\overline{x} - \sum_{i=\text{M,R,C}} t_i - \sum_{j=\text{M,R}} C_j'(\overline{x}) = 0$.

Proof: See Appendix. □

Note that the final goods supply and price in this case are equal to those in the case of forward proposing, i.e., $\mathbf{x}^* = \mathbf{X}^* = \mathbf{x}^{**} = \mathbf{X}^{**} = \overline{x}$ and $\mathbf{P}^* = \mathbf{P}^{**} = a - b\overline{x} - t_C$.

4 Tax incidence

Suppose that M and R are integrated vertically. The monopolist MR supplies final goods to competitive consumers. The cost function is $C_{MR}(X) := C_M(X) + C_R(X)$. The producer's problem is

$$\max_X \left(a - bX - \sum_{i=\text{M,R,C}} t_i\right) X - C_{MR}(X).$$

Let X_{MR} be the unique solution and let $P_{MR} = a - bX_{MR} - t_C$ be the equilibrium final goods price. Let $\pi_{MR} = \left(a - bX_{MR} - \sum_{i=\mathrm{M,R,C}} t_i\right) X_{MR} - C_{MR}(X_{MR})$ be the equilibrium profit.

Corollary 1 (Neutrality): *Let $((\mathbf{p}^*, \mathbf{x}^*), \mathbf{r}^*, (\mathbf{P}^*, \mathbf{X}^*))$ and $((\mathbf{p}^{**}, \mathbf{x}^{**}), \mathbf{r}^{**}, (\mathbf{P}^{**}, \mathbf{X}^{**}))$ be the equilibrium paths in a SCG with forward proposing and a SCG with backward proposing respectively. Similarly, let π_j^* and π_j^{**} be the respective equilibrium payoffs for $j = \{\mathrm{M,R}\}$. Production, final goods price, and joint profits are equivalent in all three cases (forward proposing, backward proposing, and a monopolist); that is,*

$$X_{MR} = \mathbf{x}^* = \mathbf{X}^* = \mathbf{x}^{**} = \mathbf{X}^{**}, \tag{3}$$
$$P_{MR} = \mathbf{P}^* = \mathbf{P}^{**},$$
$$\pi_{MR} = \pi_M^* + \pi_R^* = \pi_M^{**} + \pi_R^{**}. \tag{4}$$

Proof: (3) and (4) follow immediately from Theorem 1 and Theorem 2. Therefore, the consumers' payments are equivalent in all three cases, i.e., $\mathbf{P}^*\mathbf{X}^* = \mathbf{P}^{**}\mathbf{X}^{**} = P_{MR}X_{MR}$. Joint production costs and tax payments in all three cases are equivalent since production levels are equivalent. □

Consumers' welfare is equivalent for all three cases since the final goods prices and purchases are equivalent. Joint profits are equivalent in all three cases since the supply chain behaves as a monopolist. Therefore, the effects of taxation on consumers' welfare are equivalent in all three cases. Taxation also decreases consumers' welfare given that this is a partial equilibrium model of a market consisting of a monopolist and competitive consumers.

Despite the equivalence of joint profits, however, the profit distribution among M and R in the case of forward proposing differs from that in the case of backward proposing. The bargaining structure matters. In other words, the bargaining power matters. In a SCG, the proposer has greater bargaining power, and is the winner.

The responder cannot gain any profit. Therefore, the effects of taxation on the profit distributions are not equivalent in the two cases. We assume that $t_i > 0$ for all i in the remainder of the paper.

Theorem 3 (Tax Incidence): *Let π_j^* and π_j^{**} be the respective equilibrium payoffs for $j = \{\mathrm{M,R}\}$ in a SCG with forward proposing and in SCG with backward proposing. Then, on the one hand, taxation decreases the stronger firm's profit just as in the case of a monopolist. On the other hand, the weaker firm's profit is independent of tax rates. That is,*

$$\begin{aligned}\frac{\partial \pi_{MR}}{\partial t_i} = \frac{\partial \pi_M^*}{\partial t_i} = \frac{\partial \pi_R^{**}}{\partial t_i} = -x,\\ \frac{\partial \pi_R^*}{\partial t_i} = \frac{\partial \pi_M^{**}}{\partial t_i} = 0,\end{aligned} \tag{5}$$

for all i where x is the equilibrium final goods supply.

Proof: See Appendix. □

5 Tax shifting

Let $((p, x), \mathrm{Y}, (P, x))$ be the equilibrium path in a SCG, which depends on tax rates (recall Theorems 1 and 2).

5.1 Gross tax shifting

The ratio of a change in price to a change in the tax rate is usually regarded as shifted tax (for example, Stiglitz (2000)). We use the following notations for simplicity:

$$G(p, t_i) := \frac{\partial p}{\partial t_i},\ G(P, t_i) := \frac{\partial P}{\partial t_i}.$$

Definition 1 (Gross Tax Shifting): Suppose that a SCG is given. Let $((p, x), \mathrm{Y}, (P,$

x)) be the equilibrium path. We define *gross tax shifting* as follows:

(i) The gross *forward* shifted tax of tax i from M to R is $G(p, t_i)$ for $i = \mathrm{M}$.

(ii) The gross *backward* shifted tax of tax i from R to M is $-\mathrm{G}(p, t_i)$ for $i = \mathrm{R}, \mathrm{C}$.

(iii) The gross *forward* shifted tax of tax i from R to C is $G(P, t_i)$ for $i = \mathrm{M}, \mathrm{R}$.

(iv) The gross *backward* shifted tax of tax i from C to R is $-G(P, t_i)$ for $i = \mathrm{C}$.

If the shifted tax is equal to 1, we say that the tax shifts *fully*. If the shifted tax is greater than 0 and less than 1 (respectively, more than 1), we say that the tax shifts *partially* (respectively, *excessively*).

The proposer who has full bargaining power is the stronger in the bargaining, and hence the tax which is levied on the proposer shifts to the responder fully and the tax which is levied on the responder does not shift at all if the "law of the stronger" holds. Similarly, R should shift the tax on consumers fully and consumers should not shift the tax on R since R is the stronger in the monopolistic market.

Definition 2 (Law of the Stronger): Let $((p, x), \mathrm{Y}, (P, x))$ be the equilibrium path in a SCG. We say that *law of the stronger* holds if the stronger shifts the tax to the weaker and the weaker does not shift the tax to the stronger, i.e.,

$$G(p, t_M) = 1,\ -G(p, t_R) = 0,\ G(P, t_R) = 1 \text{ and } -G(P, t_C) = 0$$

in a SCG with forward proposing, and

$$G(p, t_M) = 0,\ -G(p, t_R) = 1,\ G(P, t_R) = 1 \text{ and } -G(P, t_C) = 0$$

in a SCG with backward proposing.

Now, does the "law of the stronger" hold? The result is negative.

Theorem 4: *The law of the stronger does not hold.*

Proof outline and interpretation: There is a precise proof in the Appendix. First, consider the case of a SCG with forward proposing (see Figure 2). Differentiating p with respect to tax rates, we have that:

$$G(p, t_M) > 0, \tag{6}$$
$$-G(p, t_R) = -G(p, t_C) = 1 - G(p, t_M), \tag{7}$$
$$0 < G(P, t_M) = G(P, t_R) < \frac{1}{2}, \tag{8}$$
$$\frac{1}{2} < -G(P, t_C) = 1 - G(P, t_M) < 1. \tag{9}$$

These equations claim that the law of the stronger does not hold. (6) and (8) show that forward shifting from the stronger to the weaker occurs. The tax which is levied on M *shifts onward*, that is, it shifts from M to R and then it also shifts from R to consumers. (9) shows that backward shifting from consumers (the weaker) to R (the stronger) occurs. This clearly violates our definition of law of the stronger and also provides negative evidence for the "law of the stronger" discussed by Mering (1942). (7) shows that whether backward shifting from R (the weaker) to M (the stronger) occurs or not is ambiguous, and therefore violates the law.

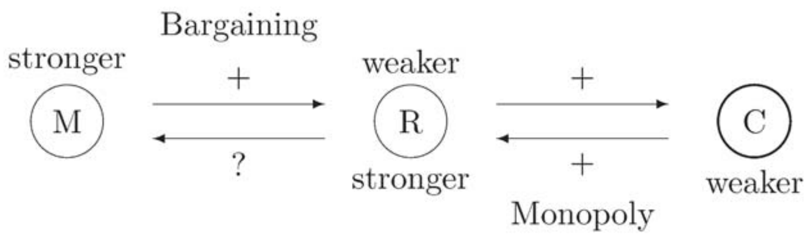

Figure 2: Gross tax shifting in a SCG with forward proposing

Second, in a SCG with backward proposing (see Figure 3), we have that:

$$G(p, t_M) = G(p, t_R) + 1, \tag{10}$$
$$-G(p, t_R) = -G(p, t_C) \geq 0, \tag{11}$$
$$0 < G(P, t_M) = G(P, t_R) < \frac{1}{2}, \tag{12}$$
$$\frac{1}{2} < -G(P, t_C) = 1 - G(P, t_M) < 1. \tag{13}$$

(12) and (13) are equivalent to (8) and (9) respectively. (13) infers backward shifting from the weaker to the stronger. (11) shows that backward shifting from R (the stronger) to M (the weaker) occurs. (11) and (13) show that the tax which is levied on consumers shifts onward and arrives at M. (10) shows that forward shifting from M (the weaker) to R (the stronger) can occur.

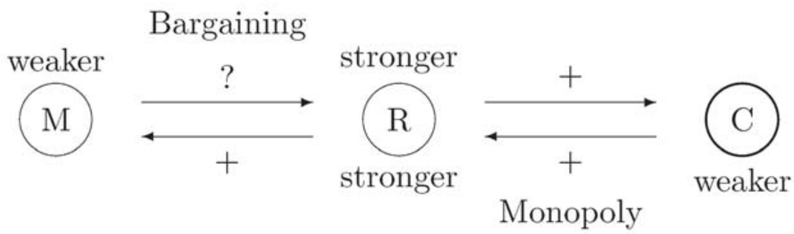

Figure 3: Gross tax shifting in a SCG with backward proposing

Finally, we conclude that the "law of the stronger" does not hold, since we have sufficient evidence which indicates that shifting from the weaker to the stronger can occur. □

The price change caused by taxation is quite ambiguous. Insofar as we observe a price change, we are unable to determine who has an advantage in the supply chain. We find that taxation decreases the stronger firm's profit even though the weaker firm's profit does not change (Theorem 3). The price change need not indicate the fact that the stronger firm suffers a loss from taxation, while the weaker loses nothing.

5.2 Net tax shifting

If an excise tax is levied, the equilibrium price of the commodity usually increases. Taxation also changes the allocation and prices of other commodities. This *environmental change* affects the r*easonable* payments and receipts among traders. The price change of the commodity is usually identified with shifted tax although it is composed of a reasonable change and a residual change, i.e., shifted tax.

Consider a hypothetical situation where the post-tax-environment prevails although the tax is *not* changed. This situation should not occur in equilibrium. It is, however, useful to resolve the actual change in price caused by taxation into a reasonable change and the shifted tax. Using a hypothetical situation (with model settings including a market structure, a bargaining structure, objective functions for traders, and so forth), we can calculate the reasonable change in price that results from an environmental change.

Consider the case of backward proposing, for example. Let (t_M, t_R, t_C) be the initial tax rates and $((p, x), r, (P, X))$ be the equilibrium path. The intermediate price p is a function of the *environment* (the allocation and prices of all commodities) and the tax rates, i.e., $p = (C_M(x)/x) + t_M x$. Suppose that the tax rates and the equilibrium path change to (t'_M, t_R, t_C) and $((p', x'), r', (P', X'))$, respectively. Using the hypothetical situation, we calculate the hypothetical price $\hat{p} = (C_M(x')/x') + t_M$, which is M's payment and R's receipt if the environment is changed but the tax rates are not. We regard $\hat{p} - p$ as a reasonable change in price, since it denotes the effect of the environmental change. We define shifted tax as the residual $p' - \hat{p}$, which is the additional price change that cannot be explained by any environmental factor.

Definition 3 (Net Tax Shifting): We define *net tax shifting* by replacing $G(k, t_i)$ with $N(k, t_i)$ for all $k = p, P$, and $i = M,R,C$ respectively in Definition 1, where

$$N(p, t_i) := \frac{\partial p}{\partial t_i} - \frac{\partial p}{\partial x}\frac{\partial x}{\partial t_i}, \; N(P, t_i) := \frac{\partial P}{\partial t_i} - \frac{\partial P}{\partial x}\frac{\partial x}{\partial t_i}.$$

Theorem 5 (Law of the Weaker): *The weaker shifts the tax to the stronger fully, though the stronger does not shift the tax at all in a SCG. In other words, let* $((p, x), Y, (P, x))$ *be the equilibrium path. We have that:*

$$N(p, t_M) = 0, \ -N(p, t_R) = \ -N(p, t_C) = 1,$$
$$N(P, t_M) = N(P, t_R) = 0, \ -N(P, t_C) = 1,$$

in a SCG with forward proposing (see Figure 4), and we also have that:

$$N(p, t_M) = 1, \ -N(p, t_R) = \ -N(p, t_C) = 0,$$
$$N(P, t_M) = N(P, t_R) = 0, \ -N(P, t_C) = 1,$$

in a SCG with backward proposing (see Figure 5).

Proof: It follows immediately from the proof of Theorem 4 in the Appendix. □

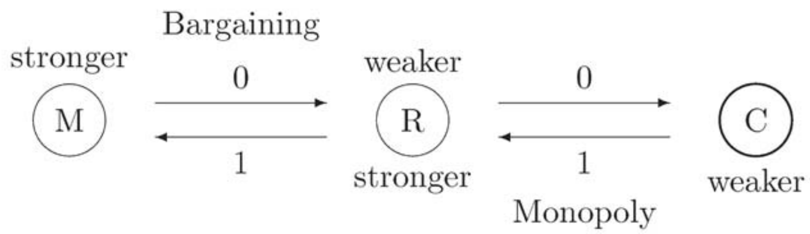

Figure 4: Net tax shifting in a SCG with forward proposing

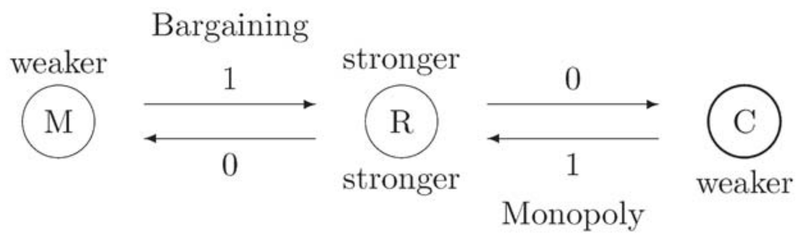

Figure 5: Net tax shifting in a SCG with backward proposing

Mering (1942, p.18) stated, "The question arises immediately why the stronger of the two has not, even before the imposition of the tax, raised the price to the highest level obtainable through his superior bargaining power." Mering (1942) contradicted the "law of the stronger". Theorem 4 supports this idea. Mering (1942, p.18) also stated, "No general rule may be set up about the extent to which the shifting of taxation is influenced by economic bargaining power." We negate this argument. Bargaining power influences tax shifting crucially though tax shifts in the opposing direction, that is, the weaker shifts the tax fully to the stronger. The responder shifts the tax fully since the responder cannot pay more tax. The responder's profit is always zero in equilibrium. The proposer does not shift the tax at all for the same reason.

Why do consumers shift the tax fully, i.e., $-N(P, t_C) = 1$? Suppose that $t_C = 0$ and let $((p, x), r, (P, X))$ be the initial equilibrium path. The final goods price is $P = a - bX$. Suppose that the tax rate increases to $t'_C > 0$ and the path changes to $((p', x'), r', (P', X'))$. The consumers' reasonable payment is $a - bX'$, which consumers pay if the environment is changed even though the tax rate is unchanged. In the monopolistic supply market, the price is quoted *on the demand curve*. The post-tax price is

$$P' = a - bX' - t'_C = \text{the reasonable payment} - (t'_C - t_C),$$

which infers full backward shifting. For the same reason, R does not shift the tax to consumers at all.

In the case of forward proposing, the tax which is levied on consumers shifts to R fully, and then also shifts from R to M. This is *onward shifting* of tax. On the contrary, the tax which is levied on M does not reach consumers.

6 Concluding remarks

We examined the effects of taxation in a situation where all bargaining power is held by one side. For example, Japanese *keiretsu* are an example of this situation. Although it may appear that the leader company shifts tax to subcontractors through superior bargaining power, we find that the subcontractors (the weaker) do not suffer from taxation. On the contrary, the authority can tax the leader company by levying a tax on the subcontractors.

We can extend the model to settings in which bargaining power is not extremely biased. We can find that the modified law of the stronger holds basically, i.e., the shifted tax from the weaker to the stronger is larger than that from the stronger to the weaker.

The number of firms is crucial for the uniqueness of the equilibrium path in a

SCG with ultimatum bargaining. If more than two firms are related vertically, the uniqueness could be assured by imposing some additional conditions or stronger equilibrium concepts.

A Appendix

A.1 Proof of Theorem 1

To verify Theorem 1, we prepare several lemmas.

Lemma 1: *Suppose that* (r^*, X^*) *is R's strategy in a SCG with forward proposing where*

$$r^* = \begin{cases} \text{Y} & \text{if } (a - bX^* - \sum_{i=\text{R,C}} t_i)X^* - C_R(X^*) - px \geq 0 \\ \text{N} & \text{if } (a - bX^* - \sum_{i=\text{R,C}} t_i)X^* - C_R(X^*) - px < 0 \end{cases}.$$

Then

$$(p^*, x^*) = \left(a - b\overline{x} - \sum_{i=\text{R,C}} t_i - \frac{C_R(\overline{x})}{\overline{x}}, \overline{x} \right) \tag{14}$$

is M's unique optimal offer strategy where $\overline{x}$ *satisfies* $a - 2b\overline{x} - \sum_{i=\text{M,R,C}} t_i - \sum_{j=\text{M,R}} C_j'(\overline{x}) = 0$.

Proof: Assumptions 1 and 2 ensure that $\overline{x}$ is unique and $0 < \overline{x} \leq \overline{X}$. First, let (p, x) be an offer which is optimal for M and is acceptable for R.

Case 1: Let $x \geq \overline{X}$. We have that

$$px = (a - b\overline{X} - \sum_{i=\text{R,C}} t_i)\overline{X} - C_R(\overline{X}),$$

since R accepts M's optimal offer (p, x) and R's production strategy is X^*. Then M's problem is

$$\max_{x \geq \overline{X}} \left(a - b\overline{X} - \sum_{i=\text{R,C}} t_i \right) \overline{X} - C_R(\overline{X}) - C_M(x) - t_M x,$$

and the solution is unique: $\overline{x} = \overline{X}$. M's payoff is positive:

$$\left(a - b\overline{X} - \sum_{i=\mathrm{M,R,C}} t_i\right)\overline{X} - \sum_{j=\mathrm{M,R}} C_j(\overline{X}) > 0 \tag{15}$$

because of Assumptions 1 and 2.

Case 2: Let $x \leq \overline{X}$. We have that

$$px = \left(a - bx - \sum_{i=\mathrm{R,C}} t_i\right)x - C_R(x),$$

and hence M's problem is

$$\max_{x \leq \overline{X}} \left(a - bx - \sum_{i=\mathrm{M,R,C}} t_i\right)x - \sum_{j=\mathrm{M,R}} C_j(x). \tag{16}$$

Taking into account the fact that $\overline{x} \leq \overline{X}$, $\overline{x}$ is the solution. Therefore, M's optimal offer is (14) and M's payoff is positive:

$$\left(a - b\overline{x} - \sum_{i=\mathrm{M,R,C}} t_i\right)\overline{x} - \sum_{j=\mathrm{M,R}} C_j(\overline{x}) > 0. \tag{17}$$

Finally, the optimal offer is (14), since (17) > (15) if $\overline{x} < \overline{X}$. Second, suppose that R rejects M's offer. M's payoff becomes zero. Such offers are never best for M because of (17). □

Lemma 1 claims that the strategy profile $((p^*, x^*), (r^*, X^*))$ is an equilibrium in a SCG with forward proposing that has an equilibrium path $\mathbb{P}^* = ((\mathbf{p}^*, \mathbf{x}^*), \mathbf{r}^*, (\mathbf{P}^*, \mathbf{X}^*))$ such that:

$$\begin{aligned}
&\textit{Proposal:} && (\mathbf{p}^*, \mathbf{x}^*) = \left(a - b\overline{x} - \textstyle\sum_{i=\mathrm{R,C}} t_i - \frac{C_R(\overline{x})}{\overline{x}}, \overline{x}\right), \\
&\textit{Response:} && \mathbf{r}^* = \mathrm{Y}, \\
&\textit{Supply:} && (\mathbf{P}^*, \mathbf{X}^*) = (a - b\overline{x} - t_C, \overline{x}).
\end{aligned}$$

Lemma 2: *Suppose that* $((p', x'), (r', X'))$ *is an equilibrium strategy profile in a SCG with forward proposing. If* $x' \neq \bar{x}$ *then M's equilibrium payoff is less than* $(a - b\overline{x} - \sum_{i=\mathrm{M,R,C}} t_i)\overline{x} - \sum_{j=\mathrm{M,R}} C_j(\overline{x})$.

Proof: Suppose that $((p', x'), (r', X'))$ is an equilibrium strategy profile such that $x' \neq \bar{x}$. We have $r' \in R^*_f$ and $X' = X^*$ from the optimality for R in Periods 2 and 3. Let $r'((p', x'))$ be a response to (p', x'). First, suppose that $r'((p', x')) = \mathrm{Y}$.

Case 1: Let $x' < \overline{X}$. We have that $\left(a - bx' - \sum_{i=\mathrm{R,C}} t_i\right) x' - C_R(x') - p'x' \geq 0$ since $r' \in R^*_f$, $X' = X^*$, and $r'((p', x')) = \mathrm{Y}$. Hence, we have that $p'x' = \left(a - bx' - \sum_{i=\mathrm{R,C}} t_i\right) x' - C_R(x')$ because of M's optimality. Then M's payoff is

$$
\begin{aligned}
p'x' - C_M(x') - t_M x' &= \left(a - bx' - \sum_{i=\mathrm{M,R,C}} t_i\right) x' - \sum_{j=\mathrm{M,R}} C_j(x') \\
&< \left(a - b\overline{x} - \sum_{i=\mathrm{M,R,C}} t_i\right) \overline{x} - \sum_{j=\mathrm{M,R}} C_j(\overline{x}).
\end{aligned}
$$

The inequality follows the definition of $\bar{x}$ that is the unique solution to the problem (16).

Case 2: Let $x' \geq \overline{X}$. Similarly, we have that $p'x' = \left(a - b\overline{X} - \sum_{i=\mathrm{R,C}} t_i\right)\overline{X} - C_R(\overline{X})$ because of M's optimality. Hence, M's payoff decreases:

$$
\begin{aligned}
p'x' - C_M(x') - t_M x' &= \left(a - b\overline{X} - \sum_{i=\mathrm{R,C}} t_i\right) \overline{X} - C_R(\overline{X}) - C_M(x') - t_M x' \\
&\leq \left(a - b\overline{X} - \sum_{i=\mathrm{M,R,C}} t_i\right) \overline{X} - \sum_{j=\mathrm{M,R}} C_j(\overline{X}) \\
&\leq \left(a - b\overline{x} - \sum_{i=\mathrm{M,R,C}} t_i\right) \overline{x} - \sum_{j=\mathrm{M,R}} C_j(\overline{x}).
\end{aligned}
$$

The first inequality is strict if $\bar{x} = \bar{X}$, while the second inequality is strict if $\bar{x} < \bar{X}$.

Second, if $r'((p', x')) = \text{N}$ then M's payoff is zero. The definition of $\bar{x}$ and the assumptions guarantee that $\left(a - b\bar{x} - \sum_{i=\text{M,R,C}} t_i\right)\bar{x} - \sum_{j=\text{M,R}} C_j(\bar{x}) > 0.$ □

Proof of Theorem 1: Suppose that $((p', x'), r', X')$ is an equilibrium path distinct from $\mathbb{P}^*$ that is supported by an strategy profile $((p', x'), (r^*, X^*))$ such that $r^* \in R_f^*$. By Lemma 2, there exists a real number $\epsilon > 0$ such that

$$(p^* - \epsilon)x^* - C_M(\bar{x}) - t_M\bar{x} > p'x' - C_M(x') - t_M x'.$$

R accepts the offer $((p^* - \epsilon), x^*)$ since

$$\begin{aligned}
&\left(a - bX^* - \sum_{i=\text{R,C}} t_i\right) X^* - C_R(X^*) - (\mathbf{p}^* - \epsilon)\mathbf{x}^* \\
&> \left(a - bX^* - \sum_{i=\text{R,C}} t_i\right) X^* - C_R(X^*) - \mathbf{p}^*\mathbf{x}^* \\
&= \left(a - b\bar{x} - \sum_{i=\text{R,C}} t_i\right) \bar{x} - C_R(\bar{x}) - \mathbf{p}^*\mathbf{x}^* \\
&= 0.
\end{aligned}$$

This is a contradiction, and hence $\mathbb{P}^*$ is the unique path if an equilibrium exists. Lastly, Lemma 1 claims that an equilibrium strategy profile exists. □

A.2 Proof of Theorem 2

Using the following two lemmas, we can verify Theorem 2 in the way similar to the proof of Theorem 1.

Lemma 3: *Suppose that r^{**} is M's strategy in a SCG with backward proposing where*

$$r^{**} = \begin{cases} \text{Y} & \text{if } px - C_M(x) - t_M x \geq 0 \\ \text{N} & \text{if } px - C_M(x) - t_M x < 0 \end{cases}.$$

Then

$$(p^{**}, x^{**}) = \left(\frac{C_M(\overline{x})}{\overline{x}} + t_M, \overline{x}\right) \tag{18}$$

is R's unique optimal offer strategy where $\overline{x}$ *satisfies* $a - 2b\overline{x} - \sum_{i=\mathrm{M,R,C}} t_i - \sum_{j=\mathrm{M,R}} C'_j(\overline{x}) = 0$.

Proof: Let (p, x) be an optimal offer. We have that $px = C_M(x) + t_M x$ and R's payoff is $(a - bX^* - \sum_{i=\mathrm{R,C}} t_i)\, X^* - C_R(X^*) - C_M(x) - t_M x$ if M accepts (p, x). Therefore, R's problem is:

$$\max_x \left(a - bX^* - \sum_{i=\mathrm{R,C}} t_i\right) X^* - C_R(X^*) - C_M(x) - t_M x.$$

Similar to the proof of Lemma 1, we have that the solution is $x = \overline{x}$ and R's payoff is positive:

$$\left(a - b\overline{x} - \sum_{i=\mathrm{M,R,C}} t_i\right) \overline{x} - \sum_{j=\mathrm{M,R}} C_j(\overline{x}) > 0.$$

□

Lemma 3 claims that the strategy profile $((p^*, x^{**}), (r^{**}, X^*))$ is an equilibrium in a SCG with forward proposing that has an equilibrium path $((\mathbf{p}^{**}, \mathbf{x}^{**}), \mathbf{r}^{**}, (\mathbf{P}^{**}, \mathbf{X}^{**}))$ such that:

$$\begin{aligned}
&\textit{Proposal:} && (\mathbf{p}^{**}, \mathbf{x}^{**}) = \left(\tfrac{C_M(\overline{x})}{\overline{x}} + t_M, \overline{x}\right), \\
&\textit{Response:} && \mathbf{r}^{**} = \mathrm{Y}, \\
&\textit{Supply:} && (\mathbf{P}^{**}, \mathbf{X}^{**}) = (a - b\overline{x} - t_C, \overline{x}).
\end{aligned}$$

Lemma 4: *Suppose that* $((p', x'), (r', X'))$ *is an equilibrium strategy profile in a SCG with backward proposing. If* $x' \neq \overline{x}$ *then M's equilibrium payoff is less than* $(a - b\overline{x} - \sum_{i=\mathrm{M,R,C}} t_i)\overline{x} - \sum_{j=\mathrm{M,R}} C_j(\overline{x})$.

Proof: Similar to the proof of Lemma 2. □

A.3 Proof of Theorem 3

Let $((\mathbf{p}^*, \mathbf{x}^*), \mathbf{r}^*, (\mathbf{P}^*, \mathbf{X}^*))$ and $((\mathbf{p}^{**}, \mathbf{x}^{**}), \mathbf{r}^{**}, (\mathbf{P}^{**}, \mathbf{X}^{**}))$ be the equilibrium paths in a SCG with forward proposing and backward proposing, respectively. By Corollary 1, the final goods supplies are equivalent in all three cases, and is denoted by x which satisfies $a - 2bx - \sum_{i=\mathrm{M,R,C}} t_i - \sum_{j=\mathrm{M,R}} C_j'(x) = 0$. Using Theorem 1, we have that

$$\pi_M^* = \mathbf{p}^*\mathbf{x}^* - C_M(\mathbf{x}^*) - t_M\mathbf{x}^* = \left(a - bx - \sum_{i=\mathrm{R,C}} t_i - \frac{C_R(x)}{x}\right) x - C_M(x) - t_M x$$
$$= \left(a - bx - \sum_{i=\mathrm{M,R,C}} t_i\right) x - \sum_{j=\mathrm{M,R}} C_j(x),$$
$$\pi_R^* = \mathbf{P}^*\mathbf{X}^* - C_R(\mathbf{X}^*) - t_R\mathbf{X}^* - \mathbf{p}^*\mathbf{x}^*$$
$$= (a - bx - t_C)\, x - C_R(x) - t_R x - \left(a - bx - \sum_{i=\mathrm{R,C}} t_i - \frac{C_R(x)}{x}\right) x = 0.$$

Using Theorem 2, we have that

$$\pi_M^{**} = \mathbf{p}^{**}\mathbf{x}^{**} - C_M(\mathbf{x}^{**}) - t_M\mathbf{x}^{**} = 0,$$
$$\pi_R^{**} = \mathbf{P}^{**}\mathbf{X}^{**} - C_R(\mathbf{X}^{**}) - t_R\mathbf{X}^{**} - \mathbf{p}^{**}\mathbf{x}^{**} = \left(a - bx - \sum_{i=\mathrm{M,R,C}} t_i\right) x - \sum_{j=\mathrm{M,R}} C_j(x).$$

In the case of vertical integration, we have that

$$\pi_{MR} = \left(a - bX_{MR} - \sum_{i=\mathrm{M,R,C}} t_i\right) X_{MR} - C_{MR}(X_{MR})$$
$$= \left(a - bx - \sum_{i=\mathrm{M,R,C}} t_i\right) x - \sum_{j=\mathrm{M,R}} C_j(x).$$

(5) follows from the fact that $\pi_R^* = \pi_M^{**} = 0$ for every $(t_M, t_R, t_C) \gg 0$ which satisfies Assumption 2. We have that

$$\frac{\partial \pi_{MR}}{\partial t_i} = \frac{\partial x}{\partial t_i}\left(a - 2bx - \sum_{i=\mathrm{M,R,C}} t_i - \sum_{j=\mathrm{M,R}} C_j'(x)\right) - x = -x.$$

□

A.4 Proof of Theorem 4

First, let x be the equilibrium final goods supply, i.e., x satisfies $a - 2bx - \sum_{i=\mathrm{M,R,C}} t_i - \sum_{j=\mathrm{M,R}} C_j'(x) = 0$. By differentiating with respect to t_i, we have that

$$-2b\frac{\partial x}{\partial t_i} - 1 - \sum_{j=\mathrm{M,R}} C_j''(x)\frac{\partial x}{\partial t_i} = 0,$$

and hence

$$\frac{\partial x}{\partial t_i} = -\frac{1}{2b + \sum_{j=\mathrm{M,R}} C_j''(x)}. \tag{19}$$

Second, let $((p, x), Y, (P, x))$ be the equilibrium path in a SCG with forward proposing, i.e., $p = a - bx - \sum_{i=\mathrm{R,C}} t_i - \frac{C_R(x)}{x}$ and $P = a - bx - t_C$. Using (19), we have that

$$\begin{aligned}\frac{\partial p}{\partial t_M} &= \frac{\partial\left(a - bx - \sum_{i=\mathrm{R,C}} t_i - \frac{C_R(x)}{x}\right)}{\partial t_M}\\ &= \frac{bx^2 + C_R'(x)x - C_R(x)}{x^2\left(2b + \sum_{j=\mathrm{M,R}} C_j''(x)\right)} > 0.\end{aligned}$$

Similarly, using (19), we have that

$$\begin{aligned}\frac{\partial p}{\partial t_R} &= \frac{\partial p}{\partial t_C} = \frac{bx^2 + C_R'(x)x - C_R(x)}{x^2\left(2b + \sum_{j=\mathrm{M,R}} C_j''(x)\right)} - 1,\\ \frac{\partial P}{\partial t_M} &= \frac{\partial P}{\partial t_R} = \frac{\partial(a - bx - t_C)}{\partial t_R} = \frac{b}{2b + \sum_{j=\mathrm{M,R}} C_j''(x)} \in \left(0, \frac{1}{2}\right),\\ \frac{\partial P}{\partial t_C} &= \frac{b}{2b + \sum_{j=\mathrm{M,R}} C_j''(x)} - 1 \in \left(-1, -\frac{1}{2}\right).\end{aligned}$$

Third, let $((p, x), Y, (P, x))$ be the equilibrium path in a SCG with backward proposing, i.e., $p = \frac{C_M(x)}{x} + t_M$ and $P = a - bx - t_C$. Using (19), we have that

$$\frac{\partial p}{\partial t_M} = \frac{\partial\left(\frac{C_M(\overline{x})}{\overline{x}} + t_M\right)}{\partial t_M} = \frac{C_M(x) - C_M'(x)x}{x^2\left(2b + \sum_{j=\mathrm{M,R}} C_j''(x)\right)} + 1,$$

$$\frac{\partial p}{\partial t_R} = \frac{\partial p}{\partial t_C} = \frac{C_M(x) - C_M'(x)x}{x^2\left(2b + \sum_{j=\mathrm{M,R}} C_j''(x)\right)} \leq 0,$$

$$\frac{\partial P}{\partial t_M} = \frac{\partial P}{\partial t_R} = \frac{\partial(a - bx - t_C)}{\partial t_R} = \frac{b}{2b + \sum_{j=\mathrm{M,R}} C_j''(x)} \in \left(0, \frac{1}{2}\right),$$

$$\frac{\partial P}{\partial t_C} = \frac{b}{2b + \sum_{j=\mathrm{M,R}} C_j''(x)} - 1 \in \left(-1, -\frac{1}{2}\right).$$

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